

Black string solutions with negative cosmological constant

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ABSTRACT: We present arguments for the existence of new black string solutions with negative cosmological constant. These higher-dimensional configurations have no dependence on the ‘compact’ extra dimension, and their conformal infinity is the product of time and $S^{d-3} \times R$ or $H^{d-3} \times R$. The configurations with an event horizon topology $S^{d-2} \times S^1$ have a nontrivial, globally regular limit with zero event horizon radius. We discuss the general properties of such solutions and, using a counterterm prescription, we compute their conserved charges and discuss their thermodynamics. Upon performing a dimensional reduction we prove that the reduced action has an effective $SL(2, R)$ symmetry. This symmetry is used to construct non-trivial solutions of the Einstein-Maxwell-Dilaton system with a Liouville-type potential for the dilaton in $(d - 1)$ -dimensions.

KEYWORDS: Black Holes, Black Holes in String Theory, AdS-CFT Correspondence.

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1. Introduction

Since its inception as a theory of gravity a tremendous amount of work has been done in General Relativity on the subject of black hole solutions of the Einstein field equations. As the end points of gravitational collapse, black holes are among the most interesting objects predicted to exist. The physics of black holes has quickly become one fascinating area of research as thermodynamics, gravity and quantum theory are intertwined in the black hole description. In recent years there has been a great deal of attention devoted towards research in black hole physics in four and higher dimensions. In more than four dimensions there are generically many available phases of black objects, with rich phase structures and interesting phase transitions between different kinds of black holes.

The physics of asymptotically Anti-de Sitter (AdS) black hole solutions is of particular interest due to the AdS/CFT conjecture. The thermodynamic properties of black holes in AdS offers the possibility of studying the nonperturbative aspects of certain conformal field theories. For example, the Hawking-Page phase transition [1] between the five dimensional spherically symmetric black holes and the thermal AdS background was interpreted by Witten, through AdS/CFT, as a thermal phase transition from a confining to a deconfining phase in the dual four dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM) theory [4]. Similarly, the phase structure of Reissner- Nordström-AdS black holes has a striking resemblance to that of van der Waals-Maxwell liquid-gas systems [5]. Also, in the presence of a negative cosmological constant Λ , so-called topological black holes have been found to exist (see [6] for reviews of the subject). For such exotic black holes, the event horizon topology is no

longer a sphere but can be any Einstein space with negative, positive or zero curvature. These results at least partially motivate attempts to find new black hole solutions with negative cosmological constant.

In this paper we present arguments for the existence of yet another class of configurations, which we interpret as the AdS counterparts of the $\Lambda = 0$ uniform black string solutions [7]. The black string solutions, present for $d \geq 5$ spacetime dimensions, exhibit new features that have no analogue in the black hole case. In the absence of a cosmological constant, the simplest vacuum static solution of this type is found by assuming translational symmetry along the extra coordinate direction and taking the direct product of the Schwarzschild solution and the extra dimension. This corresponds to a uniform black string with horizon topology $S^{d-3} \times S^1$. Though this solution exists for all values of the mass, it is unstable below a critical value as first shown by Gregory and Laflamme [8]. A branch of non-uniform black string solutions, depending also on the extra dimension was found in [9, 10] (see also the recent work [11]). The five dimensional AdS counterparts of such uniform black strings have been discussed in a more general context in a recent paper [12]. There are also known exact solutions for magnetically charged black strings in five-dimensional AdS backgrounds [13]. Their properties have been further discussed in [14] and they have been extended to higher dimensions in [15]. However, for these solutions the magnetic charge of the black strings depends non-trivially on the cosmological constant and their limit (if any) in which the magnetic charge is sent to zero in order to recover the uncharged black strings in AdS is still unknown. Other interesting solutions whose boundary topology is a fibre bundle $S^1 \times S^1 \hookrightarrow S^2$ have been found in [16] and later generalised to higher dimensions in [17].¹

Here we generalise the black string configurations of ref. [12] to higher dimensions, finding also topological black string solutions with the $(d-3)$ -dimensional sphere S^{d-3} being replaced by a $(d-3)$ -dimensional hyperbolic space. The solutions with $S^{d-3} \times S^1$ topology of the event horizon have a nontrivial zero event horizon limit. We examine the general properties of these configurations solutions and compute their mass, tension and action by using a counterterm prescription. More generally, we can replace the $(d-3)$ -dimensional angular sector with any Einstein space with positive/negative curvature leading to solutions with non-trivial boundary topologies.

By dimensionally reducing our black string solutions we find non-trivial black hole solutions of the Einstein-Dilaton system with a Liouville potential for the dilaton. We also prove that the reduced action has an effective $SL(2, \mathbb{R})$ -symmetry. We use this symmetry to generate new charged solutions of the Einstein-Maxwell-Dilaton equations with a Liouville potential. We also compare our solutions with previously known charged black hole solutions.

Our paper is structured as follows: in the next section we explain the model and derive the basic equations, while in section 3 we present a computation of the physical quantities of the solutions such as mass, tension and action. The general properties of the solutions

¹Other locally asymptotically AdS geometries with non-trivial boundary geometries and topologies can be found in [18].

are presented in section 4 where we show results obtained by numerical calculations. In the following sections we consider the dimensional reduction of our solutions to $(d - 1)$ -dimensions, proving the $SL(2, \mathbb{R})$ -symmetry of the reduced action and using it explicitly to generate charged solutions of the Einstein-Maxwell-Dilaton system with Liouville potential. We give our conclusions and remarks in the final section.

2. The model

2.1 Action principle and field equations

We start with the following action principle in d -spacetime dimensions

$$I_0 = \frac{1}{16\pi G} \int_{\mathcal{M}} d^d x \sqrt{-g} (R - 2\Lambda) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^{d-1} x \sqrt{-\gamma} K, \quad (2.1)$$

where $\Lambda = -(d - 1)(d - 2)/(2\ell^2)$ is the cosmological constant.

We consider the following parametrization of the d -dimensional line element (with $d \geq 5$)

$$ds^2 = a(r)dz^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_{k,d-3}^2 - b(r)dt^2 \quad (2.2)$$

where the $(d-3)$ -dimensional metric $d\Sigma_{k,d-3}^2$ is

$$d\Sigma_{k,d-3}^2 = \begin{cases} d\Omega_{d-3}^2 & \text{for } k = +1 \\ \sum_{i=1}^{d-3} dx_i^2 & \text{for } k = 0 \\ d\Xi_{d-3}^2 & \text{for } k = -1, \end{cases} \quad (2.3)$$

where $d\Omega_{d-3}^2$ is the unit metric on S^{d-3} . By H^{d-3} we will understand the $(d-3)$ -dimensional hyperbolic space, whose unit metric $d\Xi_{d-3}^2$ can be obtained by analytic continuation of that on S^{d-3} . The direction z is periodic with period L .

The Einstein equations with a negative cosmological constant imply that the metric functions $a(r)$, $b(r)$ and $f(r)$ are solutions of the following equations:

$$f' = \frac{2k(d-4)}{r} + \frac{2(d-1)r}{\ell^2} - \frac{2(d-4)f}{r} - f \left(\frac{a'}{a} + \frac{b'}{b} \right), \quad (2.4)$$

$$b'' = \frac{(d-3)(d-4)b}{r^2} - \frac{(d-3)(d-4)kb}{r^2 f} - \frac{(d-1)(d-4)b}{\ell^2 f} + \frac{(d-3)ba'}{ra} \\ + \frac{(d-4)b'}{r} - \frac{(d-4)kb'}{rf} - \frac{(d-1)rb'}{\ell^2 f} + \frac{a'b'}{2a} + \frac{b'^2}{b}, \quad (2.5)$$

$$\frac{a'}{a} = 2 \frac{b[\ell^2(d-3)(d-4)(k-f) + (d-1)(d-2)r^2] - (d-3)r\ell^2 f b'}{r\ell^2 f [rb' + 2(d-3)b]} \quad (2.6)$$

For $k = 0$, the Einstein equations admit the exact solution $a = r^2$, $f = 1/b = -2M/r^{d-3} + r^2/\ell^2$. This $k = 0$ solution appears to be unique, corresponding to the known planar topological black hole, respectively to the AdS soliton [19] (after an analytic continuation of the coordinates). Therefore in the remainder of this paper we will concentrate on the $k = \pm 1$ cases only.

2.2 Asymptotics

We consider solutions of the above equations whose boundary topology is the product of time and $S^{d-3} \times S^1$, $R^{d-3} \times S^1$ or $H^{d-3} \times S^1$. For even d , the solution of the Einstein equations admits at large r a power series expansion of the form:

$$\begin{aligned}
 a(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-4)/2} a_j \left(\frac{\ell}{r}\right)^{2j} + c_z \left(\frac{\ell}{r}\right)^{d-3} + O(1/r^{d-2}), \\
 b(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-4)/2} a_j \left(\frac{\ell}{r}\right)^{2j} + c_t \left(\frac{\ell}{r}\right)^{d-3} + O(1/r^{d-2}), \\
 f(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-4)/2} f_j \left(\frac{\ell}{r}\right)^{2j} + (c_z + c_t) \left(\frac{\ell}{r}\right)^{d-3} + O(1/r^{d-2}),
 \end{aligned} \tag{2.7}$$

where a_j, f_j are constants depending on the index k and the spacetime dimension only. Specifically, we find

$$a_0 = \left(\frac{d-4}{d-3}\right)k, \quad a_1 = \frac{(d-4)^2 k^2}{(d-2)(d-3)^2(d-5)}, \quad a_2 = -\frac{(d-4)^3(3d^2 - 23d + 26)k^3}{3(d-2)^2(d-3)^3(d-5)(d-7)}, \tag{2.8}$$

$$f_0 = \frac{k(d-1)(d-4)}{(d-2)(d-3)}, \quad f_1 = 2a_1, \quad f_2 = -\frac{2(d-4)^3(d^2 - 8d + 11)k^3}{(d-2)^2(d-3)^3(d-5)(d-7)}, \tag{2.9}$$

their expression becoming more complicated for higher j , with no general pattern becoming apparent.

The corresponding expansion for odd values of the spacetime dimension is given by:

$$\begin{aligned}
 a(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-5)/2} a_j \left(\frac{\ell}{r}\right)^{2j} + \zeta \log\left(\frac{r}{\ell}\right) \left(\frac{\ell}{r}\right)^{d-3} + c_z \left(\frac{\ell}{r}\right)^{d-3} + O\left(\frac{\log r}{r^{d-1}}\right), \\
 b(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-5)/2} a_j \left(\frac{\ell}{r}\right)^{2j} + \zeta \log\left(\frac{r}{\ell}\right) \left(\frac{\ell}{r}\right)^{d-3} + c_t \left(\frac{\ell}{r}\right)^{d-3} + O\left(\frac{\log r}{r^{d-1}}\right), \\
 f(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-5)/2} f_j \left(\frac{\ell}{r}\right)^{2j} + 2\zeta \log\left(\frac{r}{\ell}\right) \left(\frac{\ell}{r}\right)^{d-3} + (c_z + c_t + c_0) \left(\frac{\ell}{r}\right)^{d-3} + O\left(\frac{\log r}{r^{d-1}}\right),
 \end{aligned} \tag{2.10}$$

where we note $\zeta = a_{(d-3)/2} \sum_{k>0} (d - 2k - 1) \delta_{d,2k+1}$, while

$$c_0 = 0 \quad \text{for } d = 5, \quad c_0 = \frac{9k^3 \ell^4}{1600} \quad \text{for } d = 7, \quad c_0 = -\frac{90625k^4 \ell^6}{21337344} \quad \text{for } d = 9. \tag{2.11}$$

For any value of d , terms of higher order in $\frac{\ell}{r}$ depend only on the two constants c_t and c_z . These constants are found numerically starting from the following expansion of the solutions near the event horizon (taken at constant $r = r_h$) and integrating the Einstein

equations towards infinity:

$$\begin{aligned}
 a(r) &= a_h + \frac{2a_h(d-1)r_h}{(d-1)r_h^2 + k(d-4)\ell^2}(r-r_h) \\
 &\quad + \frac{a_h(d-1)^2r_h^2}{[(d-1)r_h^2 + k(d-4)\ell^2]^2}(r-r_h)^2 + O(r-r_h)^3, \\
 b(r) &= b_1(r-r_h) - \frac{b_1(d-4)[(d-1)r_h^2 + (d-3)k\ell^2]}{2(d-1)r_h^3 + 2(d-4)kr_h\ell^2}(r-r_h)^2 + O(r-r_h)^3, \\
 f(r) &= \frac{1}{r_h\ell^2}[(d-1)r_h^2 + k(d-4)\ell^2](r-r_h) \\
 &\quad - \frac{(d-4)}{2r_h^2\ell^2}[(d-1)r_h^2 + k(d-3)\ell^2](r-r_h)^2 + O(r-r_h)^3,
 \end{aligned}
 \tag{2.12}$$

in terms of two parameters a_h, b_1 . The condition for a regular event horizon is $f'(r_h) > 0, b'(r_h) > 0$. In the $k = -1$ case, this implies the existence of a minimal value of r_h , *i.e.* for a given Λ :

$$r_h > \ell\sqrt{(d-4)/(d-1)}. \tag{2.13}$$

Globally regular solutions with $r_h = 0$ exist for $k = 1$ only. The corresponding expansion near origin $r = 0$ is:

$$\begin{aligned}
 a(r) &= \bar{a}_0 + \frac{\bar{a}_0(d-1)}{(d-2)}\left(\frac{r}{\ell}\right)^2 + \frac{\bar{a}_0(d-1)^2}{d(d-2)^2(d-3)}\left(\frac{r}{\ell}\right)^4 + O(r^6), \\
 b(r) &= \bar{b}_0 + \frac{\bar{b}_0(d-1)}{(d-2)}\left(\frac{r}{\ell}\right)^2 + \frac{\bar{b}_0(d-1)^2}{d(d-2)^2(d-3)}\left(\frac{r}{\ell}\right)^4 + O(r^6), \\
 f(r) &= 1 + \frac{(d-1)(d-4)}{(d-2)(d-3)}\left(\frac{r}{\ell}\right)^2 + \frac{2(d-1)^2}{d(d-2)^2(d-3)}\left(\frac{r}{\ell}\right)^4 + O(r^6),
 \end{aligned}
 \tag{2.14}$$

\bar{a}_0, \bar{b}_0 being positive constants.

3. The properties of the solutions

As with the asymptotically flat case, one expects the values of mass and tension to be encoded in the constants c_t and c_z which appear in (2.7), (2.10). However, in the presence of a non-vanishing cosmological constant, the generalization of Komar's formula is not straightforward and it requires the further subtraction of a contribution from the background configuration in order to render finite results when computing the conserved charges.

While for $k = 1$ one may subtract the globally regular configuration contribution, there is no obvious choice for such a background in the $k = -1$ case. Therefore we prefer to use a different approach and follow the general procedure proposed by Balasubramanian and Kraus [20] to compute the conserved quantities for a spacetime with negative cosmological constant. This technique was inspired by the AdS/CFT correspondence and consists of adding to the action suitable boundary counterterms I_{ct} , which are functionals only of

curvature invariants of the induced metric on the boundary. Such terms will not interfere with the equations of motion because they are intrinsic invariants of the boundary metric. By choosing appropriate counterterms, which cancel the divergences, one can then obtain well-defined expressions for the action and the energy momentum of the spacetime. Unlike the background subtraction methods, this procedure is intrinsic to the spacetime of interest and it is unambiguous once the counterterm action is specified.

Thus we have to supplement the action in (2.1) with [20, 21]:

$$\begin{aligned}
 I_{\text{ct}}^0 = & \frac{1}{8\pi G} \int d^{d-1}x \sqrt{-\gamma} \left\{ -\frac{d-2}{\ell} - \frac{\ell \Theta(d-4)}{2(d-3)} R - \frac{\ell^3 \Theta(d-6)}{2(d-3)^2(d-5)} \left(R_{ab} R^{ab} - \frac{d-1}{4(d-2)} R^2 \right) \right. \\
 & + \frac{\ell^5 \Theta(d-8)}{(d-3)^3(d-5)(d-7)} \left(\frac{3d-1}{4(d-2)} R R^{ab} R_{ab} - \frac{d^2-1}{16(d-2)^2} R^3 \right. \\
 & \left. \left. - 2R^{ab} R^{cd} R_{acbd} - \frac{d-1}{4(d-2)} \nabla_a R \nabla^a R + \nabla^c R^{ab} \nabla_c R_{ab} \right) + \dots \right\}, \tag{3.1}
 \end{aligned}$$

where R and R^{ab} are the curvature and the Ricci tensor associated with the induced metric γ . The series truncates for any fixed dimension, with new terms entering at every new even value of d , as denoted by the step-function ($\Theta(x) = 1$ provided $x \geq 0$, and vanishes otherwise).

However, given the presence of $\log(r/\ell)$ terms in the asymptotic expansions (2.10) (for odd d), the counterterms (3.1) regularise the action for even dimensions only. For odd values of d , we have to add the following extra terms to (2.1) [22]:

$$\begin{aligned}
 I_{\text{ct}}^s = & \frac{1}{8\pi G} \int d^{d-1}x \sqrt{-\gamma} \log\left(\frac{r}{\ell}\right) \left\{ \delta_{d,5} \frac{\ell^3}{8} \left(\frac{1}{3} R^2 - R_{ab} R^{ab} \right) \right. \\
 & \left. - \frac{\ell^5}{128} \left(R R^{ab} R_{ab} - \frac{3}{25} R^3 - 2R^{ab} R^{cd} R_{acbd} - \frac{1}{10} R^{ab} \nabla_a \nabla_b R + R^{ab} \square R_{ab} - \frac{1}{10} R \square R \right) \delta_{d,7} + \dots \right\}.
 \end{aligned}$$

For the Kerr-AdS [23] class of higher-dimensional rotating black holes in spaces with negative cosmological constant, these terms vanish. However we shall see that they contribute non-trivially for the solutions we obtain.

Using these counterterms in odd and even dimensions, one can construct a divergence-free boundary stress tensor from the total action $I = I_0 + I_{\text{ct}}^0 + I_{\text{ct}}^s$ by defining a boundary stress-tensor:

$$T_{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta I}{\delta \gamma^{ab}}.$$

Thus a conserved charge

$$\Omega_\xi = \oint_\Sigma d^{d-2} S^a \xi^b T_{ab}, \tag{3.2}$$

can be associated with a closed surface Σ (with normal n^a), provided the boundary geometry has an isometry generated by a Killing vector ξ^a [24]. If $\xi = \partial/\partial t$ then Ω is the conserved mass/energy M . Similar to the $\Lambda = 0$ case [25], there is also a second charge associated with $\partial/\partial z$, corresponding to the solution's tension \mathcal{T} .

The computation of T_{ab} is straightforward and we find the following expressions for mass and tension:

$$M = M_0 + M_c^{(k,d)}, \quad M_0 = \frac{\ell^{d-4}}{16\pi G} [c_z - (d-2)c_t] LV_{k,d-3}, \quad (3.3)$$

$$\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_c^{(k,d)}, \quad \mathcal{T}_0 = \frac{\ell^{d-4}}{16\pi G} [(d-2)c_z - c_t] V_{k,d-3}, \quad (3.4)$$

where $V_{k,d-3}$ is the total area of the angular sector. Here $M_c^{(k,d)}$ and $\mathcal{T}_c^{(k,d)}$ are Casimir-like terms which appear for an odd spacetime dimension only,

$$M_c^{(k,d)} = -L\mathcal{T}_c^{(k,d)} = \frac{\ell^{d-4}}{16\pi G} V_{k,d-3} L \left(\frac{1}{12} \delta_{d,5} - \frac{333}{3200} \delta_{d,7} + \dots \right). \quad (3.5)$$

We note that the considered Lorentzian solutions extremize also the Euclidean action since the analytic continuation $t \rightarrow i\tau$ has no effect at the level of the equations of motion. The Hawking temperature of these solutions is computed by demanding regularity of the Euclideanized manifold as $r \rightarrow r_h$:

$$T_H = \frac{1}{4\pi} \sqrt{\frac{b_1}{r_h \ell^2} [(d-1)r_h^2 + k(d-4)\ell^2]}. \quad (3.6)$$

Thus we can proceed further by formulating gravitational thermodynamics via the Euclidean path integral [26]

$$Z = \int D[g] D[\Psi] e^{-I[g,\Psi]} \simeq e^{-I},$$

Here, $D[g]$ is a measure on the space of metrics g , $D[\Psi]$ a measure on the space of matter fields Ψ , $I[g, \Psi]$ is the action in terms of the metrics and matter fields and one integrates over all metrics and matter fields between some given initial and final Euclidean hypersurfaces, taking τ to have a period $\beta = 1/T_H$. Semiclassically the result is given by the classical action evaluated on the equations of motion, and yields to this order an expression for the entropy:

$$S = \beta M - I, \quad (3.7)$$

upon application of the quantum statistical relation to the partition function.

To evaluate the black string action, we integrate the Killing identity $\nabla^\mu \nabla_\nu \zeta_\mu = R_{\nu\mu} \zeta^\mu$, for the Killing vector $\zeta^\mu = \delta_t^\mu$, together with the Einstein equation $R_t^t = (R - 2\Lambda)/2$. Thus, we isolate the bulk action contribution at infinity and at $r = 0$ or $r = r_h$. The divergent contributions given by the surface integral term at infinity are also canceled by $I_{\text{surface}} + I_{ct}$. Together with (3.7), we find $S = A_H/4G$, where $A_H = r_h^{d-3} V_{k,d-3} L \sqrt{a_h}$ is the event horizon area. The same approach applied to the Killing vector $\zeta^\mu = \delta_z^\mu$ yields the result:

$$I = -\beta \mathcal{T} L. \quad (3.8)$$

The relations (3.7) and (3.8) lead to a simple Smarr-type formula, relating quantities defined at infinity to quantities defined at the event horizon:

$$M + \mathcal{T} L = T_H S, \quad (3.9)$$

(note that the corresponding relation in the $\Lambda = 0$ case is d -dependent [25]).

This relation also provides a useful check of the accuracy of the numerical solutions we obtain. We see now that in the limit of zero event horizon radius, the absolute values of the mass of solutions per unit length of the extra-dimension z and the tension are equal.

4. Numerical results

Starting from the expansion (2.12) and using a standard ordinary differential equation solver, we integrated the system (2.4)–(2.6) adjusting for fixed shooting parameters and integrating towards $r \rightarrow \infty$. The integration stops when the asymptotic limit (2.7), (2.10) is reached. Given (k, d, Λ, r_h) , solutions with the right asymptotics exist for one set of the shooting parameters (a_h, b_1) only.

The results we present here are found for $\ell = 1$. However, the solutions for any other values of the cosmological constant are easily found by using a suitable rescaling of the $\ell = 1$ configurations. Indeed, to understand the dependence of the solutions on the cosmological constant, we note that the Einstein equations (2.4)–(2.6) are left invariant by the transformation:

$$r \rightarrow \bar{r} = \lambda r, \quad \ell \rightarrow \bar{\ell} = \lambda \ell. \quad (4.1)$$

Therefore, starting from a solution corresponding to $\ell = 1$ one may generate in this way a family of $\ell \neq 1$ vacuum solutions, which are termed “copies of solutions“ [27]. The new solutions have the same length in the extra-dimension. Their relevant properties, expressed in terms of the corresponding properties of the initial solution, are as follows:

$$\bar{r}_h = \lambda r_h, \quad \bar{\Lambda} = \Lambda/\lambda^2, \quad \bar{T}_H = T_H/\lambda, \quad \bar{M} = \lambda^{d-4}M, \quad \text{and} \quad \bar{\mathcal{T}} = \lambda^{d-4}\mathcal{T}. \quad (4.2)$$

Now, given the full spectrum of solutions for $\ell = 1$ (with $r_{\min} < r_h < \infty$), one may find the corresponding branches for any value of $\Lambda < 0$.

Thus these solutions do not approach the uniform black string configurations as $\Lambda \rightarrow 0$ (note also that the local mass/tension of the $\Lambda = 0$ black strings decay as $1/r^{d-4}$, whereas here the decay is $1/r^{d-3}$).

We have found asymptotically AdS numerical solutions in all dimensions between five and twelve. We conjecture that they exist for any d and, in the case $k = 1$, for any value of the event horizon r_h . For $d = 5$, $k = 1$ our findings are in very good agreement with those presented in ref. [12].

The $k = 1$ solutions have a nontrivial zero event horizon radius limit corresponding to AdS vortices. As $r_h \rightarrow 0$ we find *e.g.* $c_t(d = 6) \simeq -0.0801$, $c_t(d = 7) \simeq -0.0439$, $c_t(d = 8) \simeq 0.0403$, $c_t(d = 9) \simeq 0.0229$, while $c_t(d = 10) \simeq -0.0246$.

In the same limit, we find $a(r) = b(r)$ with a nonzero value $a(0) = \bar{a}_0$, *e.g.* $\bar{a}_0(d = 6) \simeq 0.744$, $\bar{a}_0(d = 7) \simeq 0.797$, $\bar{a}_0(d = 8) \simeq 0.8316$, $\bar{a}_0(d = 9) \simeq 0.8561$, while $\bar{a}_0(d = 10) \simeq 0.8743^2$. This assigns a nonzero mass and tension to the globally regular solutions according to (3.3).

²The appearance of these values in the expansion at the origin suggests that analytic vortex solutions, if they exist, should be sought using a set of variables other than (a, b, f) .

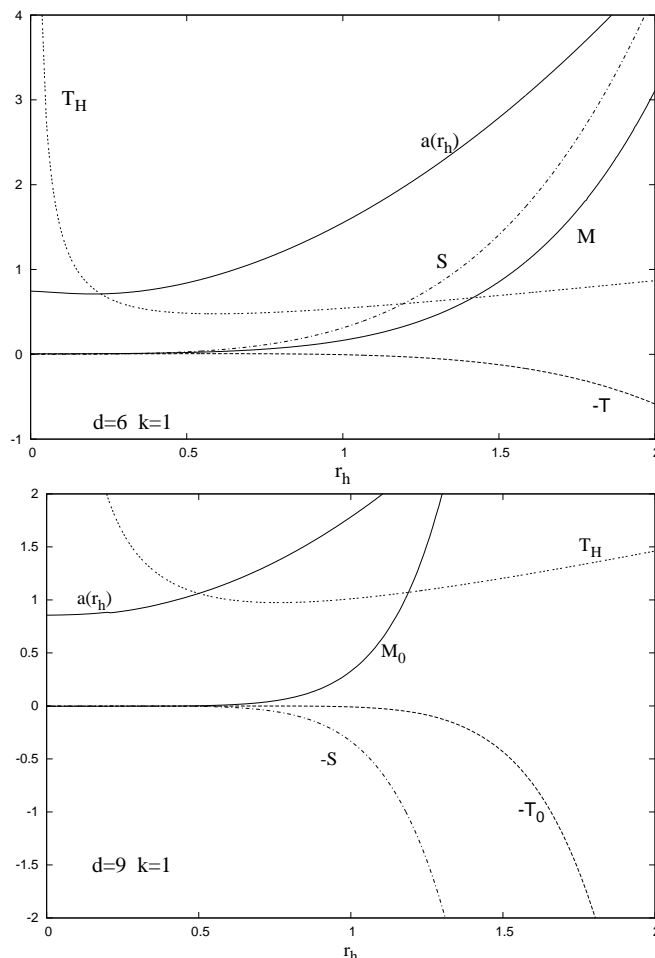


Figure 1: The mass-parameter M , the tension \mathcal{T} , (without the Casimir terms) the value of the metric function $a(r)$ at the event horizon as well as the Hawking temperature T_H and the entropy S of $k = 1$ black string solutions are represented as functions of the event horizon radius in $d = 6$, respectively $d = 9$ dimensions. In the latter diagram we plot $-S$, $-T_0$ so that they can be easily distinguished from the curve for M_0 .

The dependence of various physical parameters on the event horizon radius is presented in figure 1 for $d = 6$ and $d = 9$ solutions with $k = 1$ and figure 2 for $k = -1$ configurations in $d = 7$ and $d = 10$ dimensions.

These plots retain the basic features of the solutions we found in other dimensions (note that we set $V_{k,d-3}L = 1$ in the expressions for the mass, tension and entropy and we subtracted the Casimir energy and tension in odd dimensions).

The mass, temperature, tension and entropy of $k = -1$ solutions increase monotonically with r_h .

For all the solutions we studied, the metric functions $a(r)$, $b(r)$ and $f(r)$ interpolate monotonically between the corresponding values at $r = r_h$ and the asymptotic values at infinity, without presenting any local extrema. As a typical example, in figure 3 the metric functions $a(r)$, $b(r)$ and $f(r)$ are shown for a $d = 6$, $k = 1$ solution with $r_h =$

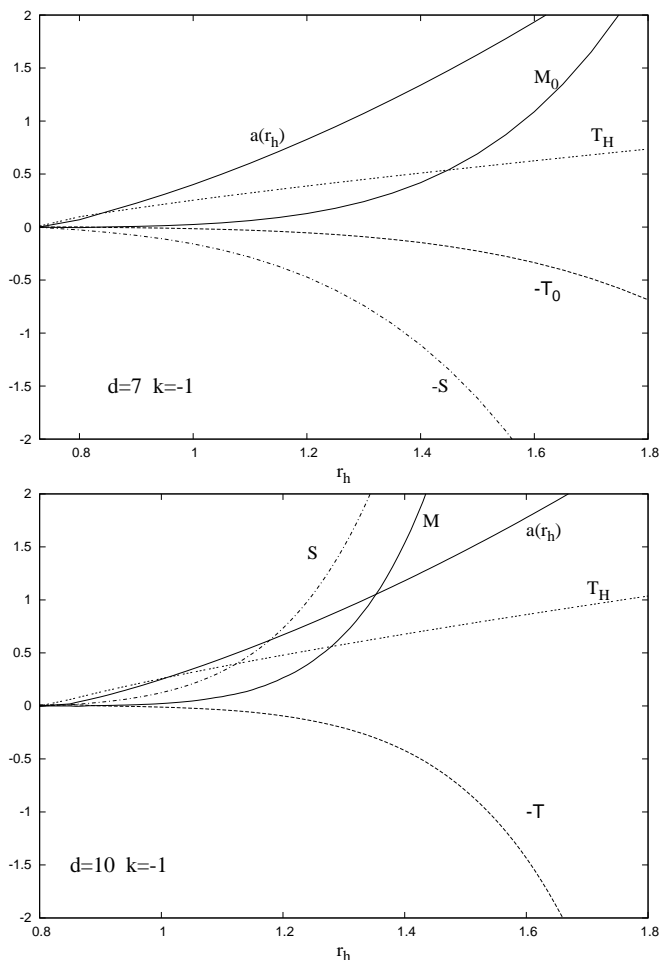


Figure 2: Same as figure 1 for $k = -1$ black string solutions in dimensions $d = 7$, respectively $d = 10$.

0.5, as functions of the radial coordinate r . One also finds numerically that the term r^2/ℓ^2 starts dominating the profile of these functions very rapidly, which implies a small difference between the metric functions for large enough r , as expected from the asymptotic expansions given in section 2.

5. $SL(2, R)$ symmetry in $(d - 1)$ -dimensions

Consider Einstein gravity coupled with a cosmological constant in d -dimensions. Its action is described by the Lagrangian:

$$\mathcal{L}_d = \tilde{e}\tilde{R} - 2\tilde{e}\Lambda, \tag{5.1}$$

where $\tilde{e} = \sqrt{-\tilde{g}}$.

Let us assume that the fields are stationary and that the system admits two commuting Killing vectors (one of them is timelike, while the other corresponds to an isometry along

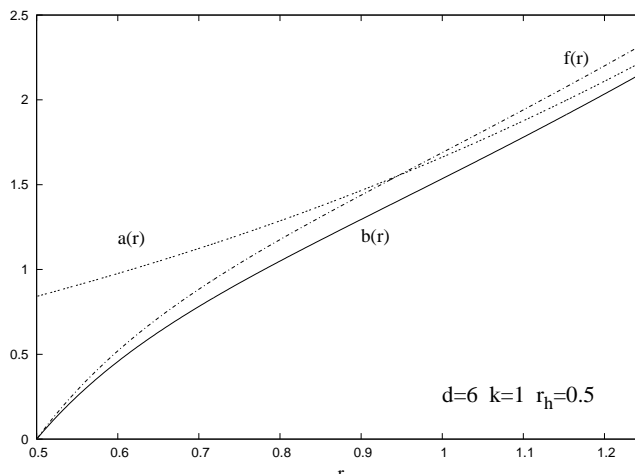


Figure 3: The profiles of the metric functions $a(r)$, $b(r)$ and $f(r)$ are shown for a typical $k = 1$ black string solution with $d = 6$, $k = 1$ and $r_h = 0.5$.

a spatial direction z). We will perform a dimensional reduction from d -dimensions down to $(d - 2)$ -dimensions along the two directions z and t . Our metric ansatz is:

$$ds_d^2 = e^{-\sqrt{\frac{2(d-3)}{d-2}}\phi} (dz + \chi dt)^2 + e^{\sqrt{\frac{2}{(d-2)(d-3)}\phi} \left[-e^{-\sqrt{\frac{2(d-4)}{d-3}}\phi_1} dt^2 + e^{\sqrt{\frac{2}{(d-3)(d-4)}\phi_1} ds_{d-2}^2 \right]}. \quad (5.2)$$

In the first step of the dimensional reduction, reducing from d to $(d - 1)$ -dimensions along the z -direction, one obtains the following $(d - 1)$ -dimensional Lagrangian³:

$$\mathcal{L}_{d-1} = eR - 2e\Lambda e^{\sqrt{\frac{2}{(d-2)(d-3)}\phi} - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{4}ee^{-\sqrt{\frac{2(d-2)}{d-3}}\phi}(\mathcal{F}_{(2)})^2, \quad (5.3)$$

while the metric and the matter fields in $(d - 1)$ -dimensions are given by the dilaton ϕ , respectively the metric and the 1-form potential:

$$ds_{d-1}^2 = -e^{-\sqrt{\frac{2(d-4)}{d-3}}\phi_1} dt^2 + e^{\sqrt{\frac{2}{(d-3)(d-4)}\phi_1} ds_{d-2}^2, \quad \mathcal{A}_{(1)} = \chi dt. \quad (5.4)$$

Here $\mathcal{F}_{(2)} = d\mathcal{A}_{(1)}$ is the 2-form field strength of the electromagnetic potential $\mathcal{A}_{(1)}$.

Performing a further dimensional reduction along the timelike direction, the Lagrangian in $(d - 2)$ -dimensions is given by:

$$\mathcal{L}_{d-2} = eR - 2e\Lambda e^{\sqrt{\frac{2}{(d-2)(d-3)}\phi} + \sqrt{\frac{2}{(d-3)(d-4)}\phi_1} - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{2}e(\partial\phi_1)^2 + \frac{1}{2}ee^{\sqrt{\frac{2(d-4)}{d-3}}\phi_1 - \sqrt{\frac{2(d-2)}{d-3}}\phi}(\partial\chi)^2.$$

³We denote $e = \sqrt{-g}$.

We shall prove next that the above Lagrangian has a global symmetry group $SL(2, \mathbb{R})$. We perform first a rotation of the scalar fields:

$$\begin{pmatrix} \hat{\phi} \\ \hat{\phi}_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{d-4}{2(d-3)}} & \sqrt{\frac{d-2}{2(d-3)}} \\ \sqrt{\frac{d-2}{2(d-3)}} & -\sqrt{\frac{d-4}{2(d-3)}} \end{pmatrix} \begin{pmatrix} \phi \\ \phi_1 \end{pmatrix}. \quad (5.5)$$

Now, let us define the matrix \mathcal{M} by:

$$\mathcal{M} = \begin{pmatrix} e^{-\hat{\phi}_1} & \chi e^{-\hat{\phi}_1} \\ \chi e^{-\hat{\phi}_1} & -e^{\hat{\phi}_1} + \chi^2 e^{-\hat{\phi}_1} \end{pmatrix}. \quad (5.6)$$

Note that $\det \mathcal{M} = -1$ hence \mathcal{M} is not an $SL(2, \mathbb{R})$ matrix. Then it is easy to see that $(d-2)$ -dimensional Lagrangian can be written in the following compact form:

$$\mathcal{L}_{d-2} = eR - 2e\Lambda e^{\frac{2\hat{\phi}}{\sqrt{(d-2)(d-4)}}} - \frac{1}{2}e(\partial\hat{\phi})^2 + \frac{1}{4}\text{etr}[\partial\mathcal{M}^{-1}\partial\mathcal{M}]. \quad (5.7)$$

The Lagrangian is manifestly invariant under $SL(2, \mathbb{R})$ transformations if one considers the following transformation laws for the potentials:

$$g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad \Lambda \rightarrow \Lambda, \quad \mathcal{M} \rightarrow \Omega^T \mathcal{M} \Omega, \quad \hat{\phi} \rightarrow \hat{\phi}. \quad (5.8)$$

where $\Omega \in SL(2, \mathbb{R})$.

6. Einstein-Maxwell-Liouville black holes

Let us apply this technique to the new black string solutions in AdS backgrounds presented in the previous sections. Starting with the d -dimensional metric (2.2) and performing a double dimensional reduction along the z and t coordinates one can read directly the fields in the $(d-2)$ -dimensional theory as:

$$\begin{aligned} ds_{d-2}^2 &= (ab)^{\frac{1}{d-4}} \left(\frac{dr^2}{f} + r^2 d\Sigma_{k,d-3}^2 \right), \\ e^{-\sqrt{\frac{2(d-3)}{d-2}}\phi} &= a, \quad e^{-\sqrt{\frac{2(d-4)}{d-3}}\phi_1} = ba^{\frac{1}{d-3}}, \quad \chi = 0. \end{aligned} \quad (6.1)$$

In order to apply the solution generating technique from the previous section, we shall perform the rotation of the scalar fields as given by (5.5). This yields

$$e^{\hat{\phi}} = (ab)^{-\frac{1}{2}\sqrt{\frac{d-2}{d-4}}}, \quad e^{\hat{\phi}_1} = \left(\frac{b}{a} \right)^{\frac{1}{2}}. \quad (6.2)$$

We are now ready to apply the $SL(2, \mathbb{R})$ -symmetry transformations as given in (5.8). For this purpose let us parameterize the matrix Ω in the form:

$$\Omega = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \quad \alpha\delta - \beta\gamma = 1. \quad (6.3)$$

From the general form of the matrix \mathcal{M} one can read the following scalar fields⁴:

$$e^{-\phi'_1} = \frac{\alpha^2 a - \gamma^2 b}{\sqrt{ab}}, \quad \chi' = \frac{\alpha\beta a - \gamma\delta b}{\alpha^2 a - \gamma^2 b}. \quad (6.4)$$

Using now the inverse transformation of the one considered in (5.5) one finds the following scalar fields corresponding to the final solution:

$$e^\phi = (\alpha^2 a - \gamma^2 b)^{-\sqrt{\frac{d-2}{2(d-3)}}}, \quad e^{\phi_1} = (ab)^{-\sqrt{\frac{d-3}{2(d-4)}}} (\alpha^2 a - \gamma^2 b)^{-\sqrt{\frac{d-4}{2(d-3)}}}. \quad (6.5)$$

Gathering all these results, one obtains in $(d-1)$ -dimensions the following fields:

$$ds_{d-1}^2 = -ab(\alpha^2 a - \gamma^2 b)^{-\frac{d-4}{d-3}} dt^2 + (\alpha^2 a - \gamma^2 b)^{\frac{1}{d-3}} \frac{dr^2}{f} + r^2 (\alpha^2 a - \gamma^2 b)^{\frac{1}{d-3}} d\Sigma_{k,d-3}^2, \\ e^\phi = (\alpha^2 a - \gamma^2 b)^{-\sqrt{\frac{d-2}{2(d-3)}}}, \quad \mathcal{A}_{(1)} = \frac{\alpha\beta a - \gamma\delta b}{\alpha^2 a - \gamma^2 b} dt. \quad (6.6)$$

which are a solution of the equations of motion derived from the Lagrangian:

$$\mathcal{L}_{d-1} = eR - 2e\Lambda e^{\sqrt{\frac{2}{(d-2)(d-3)}}\phi} - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{4}ee^{-\sqrt{\frac{2(d-2)}{d-3}}\phi}(\mathcal{F}_{(2)})^2, \quad (6.7)$$

which corresponds to an Einstein-Maxwell-Dilaton theory with a Liouville potential for the dilaton.

As a consistency check of our final solution, let us notice that if one takes $\Omega = I_2$ then one obtains the initial black string solution (2.2). Also, if $\alpha = \delta = \cosh p$ and $\beta = \gamma = \sinh p$ the effect of the $SL(2, \mathbb{R})$ transformation is equivalent to a boost of the initial black string solution in the z direction.

In general, for a generic Kaluza-Klein dimensional reduction, if the isometry generated by the Killing vector $\frac{\partial}{\partial z}$ has fixed points then the dilaton ϕ will diverge and the $(d-1)$ -dimensional metric will be singular at those points. However, this is not the case for our initial black string solutions and therefore the $(d-1)$ -dimensional fields are non-singular in the near-horizon limit $r \rightarrow r_h$. Indeed, in the near horizon limit $b(r) \rightarrow 0$ while $a(r) \rightarrow a_0$ and the $(d-1)$ -dimensional fields are non-singular. The situation changes when we look in the asymptotic region. Recall from (5.2) that

$$g_{zz} \equiv e^{-\sqrt{\frac{2(d-3)}{d-2}}\phi} = (\alpha^2 a - \gamma^2 b) \quad (6.8)$$

gives the radius squared of the z -direction in d -dimensions and that it diverges in the large r limit. Then for generic values of the parameters in Ω we find that $g_{zz} \sim r^2$ and the dilaton field in $(d-1)$ -dimensions will diverge in the asymptotic region. Physically, this means that the spacetime decompactifies at infinity; the higher-dimensional theory should be used when describing such black holes in these regions. It is amusing to note that in this limit even though the Liouville potential goes rapidly to zero, the asymptotic structure of the $(d-1)$ -dimensional metric is still non-standard. On the other hand one can choose the

⁴In what follows we shall drop the prime symbol from the fields.

parameters in Ω such that $\alpha = \gamma$. In this case the asymptotic behaviour of the $(d - 1)$ -dimensional fields is quite different as $g_{zz} \sim \frac{c_z - c_t}{r^{d-3}}$ and the radius of the z direction collapses to zero asymptotically. However the dilaton field still diverges at infinity.

Note also that starting with the regular solution in d -dimensions one obtains a globally regular $(d - 1)$ -dimensional configuration upon applying the above solution-generating procedure. This is a solution of the Einstein-Dilaton system only, with a Liouville potential term for the dilaton. Indeed, starting with the near-origin expansion given in (2.14) it is easy to see that the $(d - 1)$ -dimensional solution (6.6) will be a globally regular solution of the Einstein-Dilaton system with a Liouville potential term of the dilaton once the condition $(\alpha^2 - \gamma^2)a_0 = 1$ is satisfied. Notice that, since in the regular solution we have $a(r) = b(r)$, then the electromagnetic gauge field $\mathcal{A}_{(1)}$ is constant and, therefore, trivial. However, this is not the case for the dilaton.

It is instructive to compare our solution of the equations of motion derived from (6.7) with the previously known exact solutions from [28] (see also [29] for the dyonic extensions). Consider for instance the solution given in eqs. (6.8) in [28]:⁵

$$\begin{aligned}
 ds_{d-1}^2 &= -U(r)dt^2 + \frac{dr^2}{U(r)} + \gamma^2 r^{\frac{2}{d-1}} d\Omega_{d-3}^2, \\
 U(r) &= r^{\frac{2(d-2)}{d-1}} \left(-\frac{(d-1)^2(d-4)}{2\gamma^2(d-3)^2} - \frac{4M(d-1)}{(d-3)\gamma^{d-3}r^{\frac{2(d-3)}{d-1}}} + \frac{Q^2(d-1)^2}{(d-3)^2\gamma^{2(d-3)}} \frac{1}{r^{\frac{4(d-2)}{d-1}}} \right), \\
 e^{\sqrt{\frac{2}{(d-2)(d-3)}}\phi} &= r^{-\frac{2}{d-1}}, \quad A_t = \frac{(d-1)Q}{(d-3)\gamma^{d-3}} \frac{1}{r^{\frac{2(d-3)}{d-1}}}, \quad \Lambda = \frac{(d-2)(d-4)}{2\gamma^2}.
 \end{aligned}$$

Lifting now the solution to d -dimensions by using (5.2), performing the coordinate transformation $R = \left(\frac{d-1}{d-3}\right)r^{\frac{d-3}{d-1}}$ and rescaling the z -coordinate to absorb a constant factor one obtains:

$$\begin{aligned}
 ds_d^2 &= R^2 \left(dz - \frac{J}{2R^2} dt \right)^2 - \left(-\tilde{M} - \frac{R^2}{l^2} + \frac{J^2}{4R^2} \right) dt^2 + \\
 &\quad + \left(-\tilde{M} - \frac{R^2}{l^2} + \frac{J^2}{4R^2} \right)^{-1} dR^2 + \gamma^2 d\Omega_{d-3}^2,
 \end{aligned}$$

where we defined:

$$J = \frac{2Q(d-1)^2}{(d-3)^2\gamma^{d-3}}, \quad \tilde{M} = \frac{4(d-1)M}{(d-3)\gamma^{d-3}}, \quad \Lambda = \frac{(d-2)}{l^2}.$$

It is now manifest that the d -dimensional solution is the direct product of the analytic continuation of the three-dimensional rotating BTZ black hole [32] with a $(d - 3)$ -dimensional sphere. Note that if one analytically continues the parameters γ , l and exchanges the sphere with the metric of the hyperboloid H^{d-3} , one obtains by dimensional reduction

⁵After canonically normalising the kinetic terms of the dilaton and the electromagnetic field, comparing the two actions one reads $n = d - 1$, $a = \sqrt{d-2}$, $b = \frac{2}{(d-3)\sqrt{d-2}}$ so that $ab = \frac{2}{n-2}$. We also set $\phi_0 = 0$ for simplicity.

a non-asymptotically flat black hole with hyperbolic topology. By contrast, in our d -dimensional black string solutions there is a non-trivial warp factor multiplying the metric element of the sphere/hyperboloid; therefore the $(d - 1)$ -dimensional black hole solution is clearly different. Similar topological black hole solutions of Einstein-Maxwell-Dilaton theory with a Liouville potential for the dilaton that arise via dimensional reduction from higher dimensions have been discussed in [30, 31].

7. Discussion

In this work we have presented arguments for the existence of a new class of solutions of Einstein gravity with negative cosmological constant. For such solutions the topological structure of the boundary is the product of time and $S^{d-3} \times R$ or $H^{d-3} \times R$ and they correspond to black strings with the horizon topology $S^{d-3} \times S^1$ or $H^{d-3} \times R$ respectively (here the black string is wrapping the S^1 circle). More generally, we can replace the $(d - 3)$ -dimensional sphere (hyperboloid) with any Einstein space with positive (negative) curvature, normalised such that its Ricci tensor is $R_{ab} = k(d - 4)g_{ab}$, with $k = 0, \pm 1$.

We expect these solutions to be relevant in the context of AdS/CFT and more generally in the context of gauge/gravity dualities. Let us consider for example the 5-dimensional black string solution. The background metric upon which the dual field theory resides is found by taking the rescaling $h_{\mu\nu} = \lim_{r \rightarrow \infty} \frac{\ell^2}{r^2} \gamma_{\mu\nu}$.

Restricting to the five-dimensional $k = \pm 1$ black strings, we find

$$ds^2 = h_{ab} dx^a dx^b = -dt^2 + dz^2 + \ell^2 d\Sigma_k^2, \tag{7.1}$$

and so the conformal boundary, where the $\mathcal{N} = 4$ SYM theory lives, is $R \times S^1 \times S^2$ for $k = 1$ or $R \times S^1 \times H^2$ for $k = -1$.

The expectation value of the stress tensor of the dual CFT can be computed using the relation [33]:

$$\sqrt{-h} h^{ab} \langle \tau_{bc} \rangle = \lim_{r \rightarrow \infty} \sqrt{-\gamma} \gamma^{ab} T_{bc}, \tag{7.2}$$

which gives

$$\begin{aligned} \langle \tau_t^t \rangle &= \frac{36c_t - 12c_z - 1}{192\pi G\ell}, & \langle \tau_z^z \rangle &= \frac{-12c_t + 36c_z - 1}{192\pi G\ell}, \\ \langle \tau_\theta^\theta \rangle &= \langle \tau_\phi^\phi \rangle = \frac{-12(c_t + c_z) + 2}{192\pi G\ell}. \end{aligned} \tag{7.3}$$

As expected, this stress tensor is finite and covariantly conserved. However it is *not* traceless and its trace is precisely equal to the conformal anomaly of the boundary CFT [22]:

$$\mathcal{A} = -\frac{2\ell^3}{16\pi G} \left(-\frac{1}{8} R_{ab} R^{ab} + \frac{1}{24} R^2 \right). \tag{7.4}$$

A similar computation performed for the seven-dimensional case leads to a boundary stress tensor whose trace matches precisely the conformal anomaly of the dual six-dimensional superconformal $(2, 0)$ theory [22, 34].

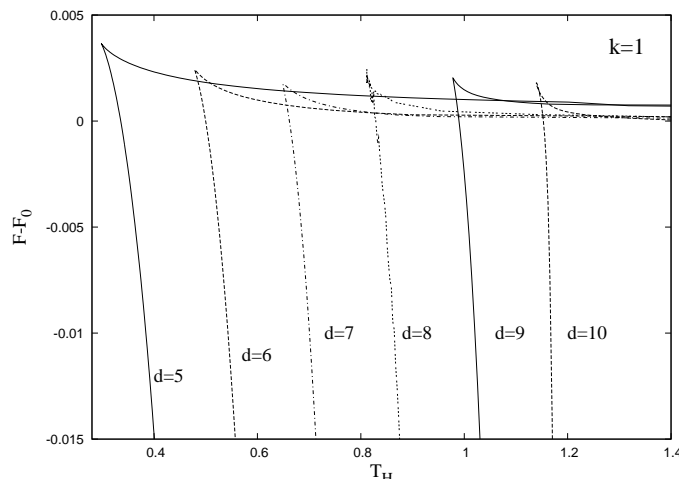


Figure 4: The free energy *vs.* the temperature for the small and large $k = 1$ black string solutions. Here we have subtracted the free energy contribution F_0 of the globally regular solution. Note that the branch corresponding to the small black holes is always positive, while the one for large black holes changes sign, rapidly becoming negative at high temperatures.

We note that the analytic continuation $z \rightarrow iu, t \rightarrow i\chi$ in the general line element (2.2) gives a bubble solution:

$$ds^2 = -a(r)du^2 + b(r)d\chi^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_{k,d-3}^2, \tag{7.5}$$

(where χ has a periodicity $\beta = 1/T_H$) whose properties can be discussed by using the methods in [12]. We find for instance a ‘small’ bubble that is the analytic continuation of the small black hole solution and also a ‘large’ bubble, which is the analytical continuation of the large black hole solution. Note that since we found that $a(r) = b(r)$ for our regular solution, its analytic continuation leads to the same regular solution. Using the counterterm approach it is possible to compute the mass of these bubble solutions with the result that:

$$M_{\text{bubble}} = -\beta T. \tag{7.6}$$

where T is given in (3.3) while $\beta = 1/T_H$, where T_H is the Hawking temperature given in (3.6). We find then that for small values of β (i.e. small size of the χ -circle at infinity) the ‘large’ bubble solution has less mass than either the regular solution or the ‘small’ bubble⁶, while for circles with size large enough the background solution has the minimum energy.

As with the spherically symmetric Schwarzschild-AdS solutions, the temperature of the $k = 1$ black string solutions is bounded from below, as we can see in figures 1 and 2. At low temperatures we have a single bulk solution, which we conjecture to correspond to the thermal globally regular solution. At high temperatures there exist two additional solutions that correspond to the small and large black holes. The free energy $F = I/\beta$ of

⁶We thank Keith Copsey and Gary Horowitz for pointing this out to us.

the $k = 1$ solutions is positive for small r_h and negative for large r_h . This suggests that the phase transition found in [1] occurs here as well. Indeed, in figure 4 we plot the free energy versus the temperature for the small and large black hole solutions for $5 \leq d \leq 10$. We observe the physics familiar from the Schwarzschild-AdS case: we have the two branches consisting of smaller (unstable) and large (stable) black holes. The entire unstable branch has positive free energy while the stable branch' free energy goes rapidly negative for all $T > T_c$. Here T_c is the critical temperature at which we observe a phase transition between the large black holes and the thermal globally regular background.

As avenues for further research, it would be interesting to consider black hole solutions with an S^{d-2} event horizon topology, whose conformal infinity is the product of time and $S^{d-3} \times S^1$, presenting the asymptotic expansion (2.7), (2.10). Such solutions are known to exist for $\Lambda = 0$, approaching asymptotically $(d - 1)$ -dimensional Minkowski space times a circle $\mathcal{M}^{d-1} \times S^1$ [35]. Also, the black string solutions may be useful in finding the AdS version of the $d = 5$ asymptotically flat black rings [36]. The heuristic construction of black rings discussed in [37] applies in this case too, and we expect an AdS black ring to approach the black string solution in the limit where the radius of the ring circle grows very large.

Another interesting issue to investigate is the Gregory-Laflamme instability [8]. We expect the black string solutions discussed in this paper to be unstable for some critical values of the parameters.

We also note that the existence of globally regular solutions and of black hole solutions suggests that there might be some kind of critical phenomena associated with the collapse of matter to the black string, in keeping with the critical phenomena encountered in the study of the gravitational collapse of matter fields in spaces with spherical symmetry (see [38] and the references therein). This is because a given distribution of matter undergoing collapse could potentially form either of these solutions, depending upon certain parameters in the initial data. At the bifurcation point one presumably would see critical phenomena. A consideration of these aspects is outside the scope of this work.

Further analysis of these metrics and their role in string theory remain interesting issues to explore in the future.

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